MA2115 Matemáticas IV (semi-presencial)
Práctica 06

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Ejemplo 1

$$ydx + \left(x - \frac{1}{2}x^3y\right)dy = 0$$

$$y\frac{dx}{dy} + x - \frac{1}{2}x^3y = 0$$

$$\frac{dx}{dy} + \frac{x}{y} = \frac{1}{2}x^3$$

$$\frac{x'}{x^3} + \frac{1}{yx^2} = \frac{1}{2}$$

$$-\frac{z'}{2} + \frac{z}{y} = \frac{1}{2}$$

$$z' - 2\frac{z}{y} = -1$$

Hacemos el cambio de variables:

$$z = x^{1-3} = \frac{1}{x^2}$$

$$z' = -2\frac{x'}{x^3}$$

Obtenemos una ecuación de Primer Orden.

$$z' - 2\frac{z}{y} = -1$$

Ejemplo 1 (Continuación)

$$z' - 2\frac{z}{y} = -1, z = \frac{1}{x^2}$$

$$\mu(y) = e^{\int P(y)dy} = e^{\int -\frac{2}{y}dy} = \frac{1}{y^2}$$

$$z = \frac{1}{\mu(y)}\left(\int \mu(y)g(y)dy + C\right) = y^2\left(\int -\frac{1}{y^2}dy + C\right)$$

$$= y^2\left(\frac{1}{y} + C\right) = y + Cy^2$$

$$\boxed{\frac{1}{x^2} = y + Cy^2}$$

Ejemplo 2

$$\frac{2ydy}{x^3} + \left(\frac{x^2 - 3y^2}{x^4}\right) dx = 0$$

$$\frac{2y}{x^3} y' + \frac{x^2 - 3y^2}{x^4} = 0$$

$$\frac{2y}{x^3} y' + \frac{1}{x^2} - \frac{3y^2}{x^4} = 0$$

$$y' + \frac{x}{2y} = \frac{3y}{2x}$$

$$y' - \frac{3}{2x} y = -\frac{x}{2} y^{-1}$$

$$2yy' - \frac{3}{x} y^2 = -x$$

$$z' - \frac{3z}{x} = -x$$

Hacemos el cambio de variables:

$$z = y^{1-(-1)} = y^2$$

$$z' = 2yy'$$

Obtenemos una ecuación de Primer Orden.

$$z' - \frac{3z}{x} = -x$$

Ejemplo 2 (Continuación)

$$z' - \frac{3z}{x} = -x, z = y^2$$

$$\mu(x) = e^{\int P(x)dx} = e^{\int -\frac{3}{x}dx} = \frac{1}{x^3}$$

$$z = \frac{1}{\mu(x)}\left(\int \mu(x)g(x)dx + C\right) = x^3\left(\int -x\frac{1}{x^3}dx + C\right)$$
$$= x^3\left(\frac{1}{x} + C\right) = x^2 + Cx^3$$

$$\boxed{y^2 = x^2 + Cx^3}$$

Ejemplo 1

$$(2x-y+7)dy+(x-2y+2)dx=0$$

Sean $u=2x-y+7$ y $v=x-2y+2$

$$x=\frac{2u-v}{3}-4 \quad \text{y} \quad y=\frac{u-2v}{3}-3$$

$$u\left(\frac{du-2dv}{3}\right)+v\left(\frac{2du-dv}{3}\right)=0$$

$$(u+2v)du-(2u+v)dv=0$$

$$\frac{dv}{du}=\frac{u+2v}{2u+v}$$

$$\frac{dv}{du}=\frac{1+2\frac{v}{u}}{2+\frac{v}{u}}$$

Ejemplo 1 (Continuación)

$$\frac{dv}{du} = \frac{1+2\frac{v}{u}}{2+\frac{v}{u}}$$

Hacemos $z = \dfrac{v}{u}$ o $v = zu$, de donde $\dfrac{dv}{du} = \dfrac{dz}{du}u + z$

Sustituyendo en la ecuación diferencial obtenemos

$$\frac{dz}{du}u + z = \frac{1+2z}{2+z} \implies \frac{dz}{du}u = \frac{1+2z}{2+z} - z$$

$$\frac{dz}{du}u = \frac{1-z^2}{2+z} \implies \frac{2+z}{1-z^2}dz = \frac{du}{u}$$

$$\left(\frac{2}{1-z^2} + \frac{1}{2}\frac{2z}{1-z^2}\right)dz = \frac{1}{u}du$$

$$\ln\left|\frac{1+z}{1-z}\right| - \frac{1}{2}\ln\left|1-z^2\right| = \ln|Au| \implies 2\ln\left|\frac{1+z}{1-z}\right| - \ln\left|1-z^2\right| = 2\ln|Au|$$

$$Cu^2 = \frac{1+z}{(1-z)^3}$$

Ejemplo 1 (Continuación)

$$Cu^2 = \frac{1+z}{(1-z)^3}, \textit{ con } z = \frac{v}{u}, u = 2x - y + 7 \textit{ y } v = x - 2y + 2$$

$$Cu^2 = \frac{1+z}{(1-z)^3}$$

$$Cu^2 = \frac{1+\frac{v}{u}}{\left(1-\frac{v}{u}\right)^3}$$

$$Cu^2 = u^2\frac{u+v}{(u-v)^3}$$

$$C(u-v)^3 = v+u$$

$$\boxed{C(x+y+5)^3 = 3x-3y+9}$$

Ejemplo 2

$$y' = \frac{1 - 6x - 3y}{1 + 2x + y}$$

$$u' - 2 = \frac{-3u + 4}{u}$$

$$u' = \frac{-u + 4}{u}$$

$$\frac{u}{4 - u} du = dx$$

$$u = 1 + 2x + y$$

$$-3u = -3 - 6x - 3y$$

$$-3u + 4 = 1 - 6x - 3y$$

$$u' = 2 + y'$$

$$\left(-1 + \frac{4}{4 - u}\right) du = dx$$

$$-u - 4\ln|4 - u| = x + A$$

$$\boxed{-2x - y - 4\ln|3 - 2x - y| = x + B}$$

Ejemplo 3

$$y' = (x+y)^2$$

Sea $u = x + y$ entonces $u' = 1 + y'$

$$u' - 1 = u^2$$

$$u' = u^2 + 1$$

$$\frac{du}{u^2+1} = dx$$

$$\arctan(u) = x + C$$

$$\boxed{\arctan(x+y) = x + C}$$

Ejemplo 1

$$y'' + (y')^2 = 0$$

Cambio de variable:

$$v = y'$$
$$v' = y''$$

$$v' + v^2 = 0$$
$$\frac{dv}{v^2} = -1\,dx$$
$$-\frac{1}{v} = -x + C$$
$$v = \frac{1}{x + C_1}$$
$$y' = \frac{1}{x + C_1}$$

$$\boxed{y = \ln(x + C_1) + C_2}$$

Ejemplo 2

$$x^2y'' + 2xy' - 1 = 0 \quad (x > 0)$$

Hacemos el cambio de variables:

$$v = y'$$
$$v' = y''$$

$$v' + \frac{2}{x}v = \frac{1}{x^2}$$

Factor integrante: $\mu(x) = \exp\left(\int \frac{2}{x}dx\right) = x^2$

$$v = \frac{1}{\mu(x)}\left(\int \mu(x)\frac{1}{x^2}dx + C_1\right)$$

$$v = \frac{1}{x^2}(x + C_1)$$

$$y' = \frac{1}{x} + \frac{C_1}{x^2}$$

$$\boxed{y = \ln(x) + \frac{C_1}{x} + C_2}$$

Ejemplo 3

$$2x^2y'' + (y')^3 = 2xy'(x > 0)$$

$$v' - \frac{1}{x}v = -\frac{1}{2x^2}v^3$$

$$-2\frac{v'}{v^3} + \frac{2}{xv^2} = \frac{1}{x^2}$$

$$z' + \frac{2}{x}z = \frac{1}{x^2}$$

Cambio de variable:

$$v = y'$$
$$v' = y''$$

Bernoulli ($\alpha = 3$):

$$z = v^{1-3} = v^{-2}$$
$$z' = -2v^{-3}v'$$

Factor integrante:

$$\mu(x) = \exp\left(\int \frac{2}{x}dx\right) = x^2$$

$$z = \frac{1}{\mu(x)}\left(\int \mu(x)\frac{1}{x^2}dx + C_1\right)$$

$$z = \frac{1}{x^2}(x + C_1)$$

Ejemplo 3 (Continuación)

$$z = \frac{x + C_1}{x^2}$$

$$v^2 = \frac{x^2}{x + C_1}$$

$$v = \frac{x}{\sqrt{x + C_1}}$$

$$y' = \frac{x}{\sqrt{x + C_1}}$$

$$x > 0$$

Cambios de variables:

$$v = y'$$

$$v' = y''$$

$$z = v^{-2}$$

$$\boxed{y = \tfrac{2}{3}(x - 2C_1)\sqrt{x + C_1} + C_2}$$

FIN